Morning Sessions

Algebra, SAS 2225

10:30 *Vertex Operators and the Kostka-Foulkes Polynomials*

Timothee William Bryan, Naihuan Jing NCSU Advisor: Naihuan Jing `twbryan@ncsu.edu`

The familiar Hall-Littlewood polynomials, $H_\mu[X;t]$ form a basis for symmetric functions and are related to the Schur function, $s_\mu[X]$, basis via

$$H_\mu[X;t] = \sum_{\lambda \vdash |\mu|} K_{\lambda\mu}(t) s_\lambda[X]$$

where $K_{\lambda\mu}$ is the Kostka-Foulkes Polynomial. Lascoux and Schützenberger proved that for semi-standard Young tableaux

$$H_\mu[X;t] = \sum_{T \in SST^\mu} t^{charge(T)} s_{shape(T)}[X]$$

where the charge of a tableau T is a value obtained by weighting the entries of a reading word corresponding to a filling using content μ in a particular fashion. We define an algebraic formula for the Kostka-Foulkes polynomials using Hall-Littlewood vertex operators and Jing's Hall-Littlewood inner product which does not utilize Lascoux and Schüteznberger's result. We will also discuss combinatorial symmetries which arise during the calculations and proof of our result.

10:55 *Twisted Logarithmic Modules of the Symplectic Fermions*

McKay Sullivan, Bojko Bakalov NCSU Advisor: Bojko Bakalov `smsulli4@ncsu.edu`

We discuss the recently defined notion of a twisted logarithmic module of a vertex algebra. One of the simplest cases of a vertex algebra admitting twisted logarithmic modules is the vertex algebra of symplectic fermions. We give explicit examples of such modules obtainable as highest weight representations on a certain Fock space. We conclude with a brief comment on the similarities between the two-dimensional case and an example from logarithmic conformal field theory.

11:20 *Classification of some solvable Leibniz algebras*

Ismail Demir NCSU Advisor: Kailash Misra `idemir@ncsu.edu`

Leibniz algebras are non-antisymmetric generalization of Lie algebras. Classification of all solvable Lie algebras is presently unsolved and is very difficult problem. Due to lack of antisymmetry in Leibniz algebras, the problem of classifying all solvable Leibniz algebras is more complicated. We give classification of solvable Leibniz algebras with one dimensional derived subalgebra. We use the canonical forms for the congruence classes of matrices of bilinear forms to obtain our result.

11:45 *Characterizing Solvable Leibniz Algebras*

Bethany Turner NCSU Advisor: Ernest Stitzinger, Kailash Misra `bnturne2@ncsu.edu`

Leibniz algebras are certain generalizations of Lie algebras. Work is underway to generalize known results for Lie algebras to analogous results in Leibniz algebras. In this talk, we give some characterizations for solvable Leibniz algebras according to the behavior of their maximal subalgebras. Specifically we discuss c-ideals, and subalgebras with the covering-avoidance property.

PDEs, SAS 2102

10:30 *Variable Importance for Scattered Data*

Joey Hart, Pierre Gremaud NCSU Advisor: Pierre Gremaud jlhart3@ncsu.edu

Many problems in science and engineering are plagued by high dimensionality that limits mathematical analysis. One way to combat this is to compute a measure of the importance of the variables and fix unimportant ones. However, in many applications the underlying function is only known through scattered data. In this case, a statistical model must be constructed and variable importance computed from it. There are a variety of statistical models and variable importance measure that may be chosen. Determining which statistical models and variable importance measures too use remains an open question. In this talk, we present a new algorithm to compute variable importance from scattered data and compare it with existing methods.

10:55 *Numerical Computation of a heteroclinic orbit based on the Principle of Wazewski*

John Jinho Kim NCSU Advisor: Xiao-Biao Lin jkim40@ncsu.edu

The principle of Wazewski states that if there is a bounding region for the differential equation $\dot{x} = f(x)$ that satisfies certain hypotheses, then there exists a solution inside the region all the time. This has been a major tool to show the existence of traveling waves in many areas of mathematics such as predator-prey models, chemical reactions, and thermodynamics. The purpose of this research is twofold. The first part is to develop a shooting method based on the principle of Wazewski to numerically compute the heteroclinic solution of the dynamical system. The second part is to prove an inverse of the principle of Wazewski for some specific cases. This talk is based on the thesis in North Carolina State University under direction of Professor Xiao-Biao Lin.

11:20 *PDE Solvers for Hybrid Architectures*

Michael Malahe, Sorin Mitran UNC-CH Advisor: Sorin Mitran mmalahe@gmail.com

Recent developments in high-performance computing have led to a hybrid architecture composed of multicore CPUs and GPU coprocessors (e.g. ORNL Titan machine). In such architectures, a premium is placed on algorithms that require minimal communication between CPU/GPU nodes. We introduce a set of algorithms in this spirit based upon stochastic differential equation solvers for kinetic level models of elliptic PDEs, coupled with high-order finite difference models of the continuum (homogenized) scale.

Biomath, SAS 1108

10:30 *Mathematical Model of Hepatitis C Viral Dynamics using a Combination Therapy of Interferon, Ribavirin, and Telaprevir*

George Bernard Lankford, Hien Tran, Phuong Hoang, Haley O'Farrell, Philip Aston, Katie Cranfield, and Alex Cassenote NCSU Advisor: Hien Tran gblankfo@ncsu.edu

Hepatitis C is a virus that affects the liver and is one of the leading causes for cirrhosis. Recently, there has been an introduction of drugs called direct acting antivirals that have improved the chance for sustained viral response from around 50% to around 90%. In this talk, we introduce a new mathematical model for Hepatitis C dynamics treated with the direct acting antiviral drug, telaprevir, alongside interferon and ribavirin. We also demonstrate the sensitivity and identifiability techniques used to validate the model, as well as fit the model to data received from clinical trials.

10:55 *Fluid-structure interaction, immersed boundary methods, muscle models, and all that jazz.*

Nicholas A Battista UNC-CH Advisor: Laura A. Miller nick.battista@unc.edu

Since their invention in the 1970s, immersed boundary methods have been applied to a wide range of fluid-structure interaction problems, across many scientific disciplines. We present an easy to use fiber model based immersed boundary software package in MATLAB that can be used for teaching purposes, research, and recreation, with the capability for easy addition of new fiber-structure models. The package itself, IB2d, contains many examples that illustrate the current depth of the open-source package, whose continual updates are found at https://github.com/nickabattista.

11:20 *Modeling CRISPR/Cas based gene drives for population replacement.*

Michael Vella, Alun Lloyd, Fred Gould NCSU Advisor: Alun Lloyd mrvella@ncsu.edu

A gene drive biases inheritance of a novel gene such that a wild-type population is completely replaced by a driven, mutant strain over many generations. A successful gene drive mechanism would have many ecological applications including the prevention of the spread of many vector-borne diseases and reversing pesticide resistance. There has been a renewed excitement regarding gene drives due to the CRISPR/Cas9 system, which provides an effective way to convert heterozygote wild-type and mutant individuals to mutant homozygotes. We illustrate difference equation models that can help understand the underlying population dynamics of the drive. We can use such models to predict the efficacy of the construct under various parameter regimes, consider potential ways to limit or reverse the drive in the case of accidental releases or releases that have unintended effects, and to estimate parameters given experimental data.

Data Processing and Sensitivity Analysis, SAS 2106

10:30 *Sensitivity Analysis: Sensitivity Equations vs. Complex-Step Method*

Marcella Noorman, H.T. Banks, Lorena Bociu, Kristen Tillman, and Kidist Zeleke NCSU Advisor: Lorena Bociu, H. T. Banks mjnoorma@ncsu.edu

Often times, we want to control certain physical processes by the parameters present in the model. A sensitivity analysis of the model unveils how the parameters affect the solution, giving way to the control problem. The classic method used to perform sensitivity analysis is to derive the sensitivity equations from the original equation and solve for the sensitivities from the resulting system. In 'An Automated Method for Sensitivity Analysis Using Complex Variables,' Martins, Kroo, and Alonso present a new method for computing sensitivities. This method, called the complex step method, uses the Cauchy-Riemann equations to approximate the sensitivities. While this is computationally more efficient than using the sensitivity equations, one must be extra careful in its implementation as it requires an analytic algorithm. In this work, we numerically estimate the sensitivities for various models using the classical method with finite elements and the complex step method. Our results show the two methods to be comparable.

10:55 *Parameter Subset Selection for Mixed-Effects Models*

Katie Schmidt NCSU Advisor: Ralph Smith klschmid@ncsu.edu

Mixed-effects models are a popular choice for describing data obtained from multiple experiments, but mixed-effects model selection remains an open area of research. Many current techniques are limited in that they are computationally prohibitive for large problems or cannot be applied to nonlinear models. To aid in model selection, we introduce a parameter subset selection (PSS) algorithm for mixed-effects models. We provide examples to verify the effectiveness of the PSS algorithm and to test the performance of mixed-effects model selection that makes use of parameter subset selection.

11:20 *The Continuous Configuration Model: Extracting Communities from Edge-Weighted Networks*

John Palowitch, Andrew B. Nobel, Shankar Bhamidi UNC-CH Advisor: Andrew Nobel palojj@ncsu.edu

Community detection is the process of grouping strongly connected nodes in a network. Few community detection methods are able to handle simultaneously the many nuances of real data, like edge weights, overlapping communities, and background nodes. In this paper, we introduce the continuous configuration model, a novel theoretical tool admitting flexible and statistically principled community detection. We prove a central limit theorem regarding edge sums from the model, and use this result to motivate a community detection method called Continuous Configuration Model Extraction (CCME). The method iteratively extracts communities via node-wise hypothesis tests, allowing for overlapping communities and the anti-detection of background. We show CCME to be comparable in speed and accuracy to other methods while enjoying advantages in Type I error in the presence of background nodes. We apply CCME to real data from social science, air traffic, and other sources.

11:45 *Degradation detection in composite materials using reflectance spectroscopy*

Jared Catenacci, H.T. Banks NCSU Advisor: H. T. Banks jwcatena@ncsu.edu

In this talk we will show that reflectance spectroscopy obtained from a thermally treated ceramic matrix composite can be used to quantity the products of oxidation. The data collection will be described in detail in order to point out the potential biasing present in the data processing. A probability distribution is imposed on select model parameters, and then non-parametrically estimated. A non-parametric estimation is chosen since the exact composition of the material is unknown due to the inherent heterogeneity of ceramic composites. We will demonstrate, using a weighted least squares estimation, that we are able to detect a distinguishable increase in the SiO2 present in the samples which were heat treated for 100 hours compared to 10 hours.

Afternoon Sessions

Topology, SAS 2225

1:15 *Analytic torsion: generalized metric invariance*

Phillip Andreae Duke Advisor: Mark Stern pandreae@math.duke.edu

We study the Ray-Singer analytic torsion T associated to a flat vector bundle with hermitian metric h over an odd-dimensional compact manifold with Riemannian metric g. In the acyclic case (and, with the appropriate interpretation, more generally), T is known to be independent of the metrics h and g, i.e., T is a topological invariant. We frame the metric independence of T in terms of a certain closed one-form on the space of metrics, and we prove that furthermore T is independent of the metric on the exterior bundle, which may be chosen independently of g.

1:40 *Structured Categories are Algebras over their String Diagram Operad*

Dmitry Vagner, David I. Spivak, Patrick Schultz Duke Advisor: Ezra Miller dv@math.duke.edu

It is well known that presenting a particular group in terms of generators and relations involves arbitrary choice. It is less well known that defining the notion of a group (in terms of a set equipped with operations that satisfy equational axioms) also involves arbitrary choice. In his 1963 doctoral thesis, William Lawvere invented a way of doing universal algebra that provided "unbiased" definitions for any algebraic object (e.g. groups, rings, associative algebras etc.). In the mid 1980's, Andre Joyal and Ross Street developed the theory of string diagrams, which allow one to do algebraic calculations in a monoidal category using purely topological intuition. It was recently realized, and only this year formalized, that one could capture many "categories with extra structure" (also called "doctrines") in a similarly unbiased way: by considering them as algebras over the operad of their graphical language. This further explores the inherently topological nature of higher category theory. In this talk, I will give a historical overview of this story and then move on to the special case of operads: in particular we will construct (and explain the meaning of) "an operad for operads."

2:05 *Noncommutative Instantons*

Sam Miller UNC-CH Advisor: Justin Sawon samill@live.unc.edu

Instantons have been an important focus of study for mathematicians and physicists since their inception. For physicists, they are seen as a minimal energy for a certain action and to a mathematician, they are connections ∇ on a four-manifold such that the curvature F_∇ obeys $*F_\nabla = \pm F_\nabla$. The moduli space of instantons on $\mathbb{R}^4$ is not complete. There is a natural completion, but this introduces singularities. Smoothing away these singularities can be done by changing a parameter but we then lose some geometric meaning. We will see that if we interpret this situation as instantons over a noncommutative $\mathbb{R}^4$ ($\mathbb{R}^4_{NC}$), we regain the geometric meaning as the moduli space of instantons over $\mathbb{R}^4_{NC}$. This follows from Nekrasov and Schwarz (1998).

2:30 *Some results in chromatic graph cohomology*

Dan Scofield NCSU Advisor: dscofie@ncsu.edu

Khovanov homology is an invariant of knots and links that categorifies the Jones polynomial. Helme-Guizon and Rong (2005) introduced a Khovanov-type, bigraded homology for graphs that categorifies the chromatic polynomial. We present several results about the occurrence of torsion and some conjectures about this homology over general algebras A_m.

PDEs, SAS 2102

1:15 *Displacement Field Calculations for Substrate with Resting two dimensional Droplet*

Aaron Bardall NCSU Advisor: Michael Shearer arbardal@ncsu.edu

A fluid droplet will cause significant displacement in the surface of a soft substrate upon which it rests. The effects that drive the displacement are the upward pull at the contact line of the droplet and the internal pressure pushing down on the substrate. This scenario has been studied for sufficiently small droplets for which gravity has little influence. Here a solution for surface pressure of a gravitationally influenced two dimensional droplet is presented, enabling the evaluation of the surface displacement of a substrate for droplets of any size. The solution technique for evaluating the surface displacement which utilizes a Fourier transform will also be discussed. The method incorporates general interfacial stress scenarios which govern the contact angle of the droplet as well as a nontrivial radial contact-line model. Results of the solution technique are presented for droplet sizes below and above the gravitationally influenced limit.

1:40 *Experimental Analysis of the Diffusion of a Passive Scalar Subject to Steady Flow in a Circular Pipe.*

Francesca Bernardi, Sarah C. Burnett, Roberto Camassa, Richard M. McLaughlin UNC-CH Advisor: Roberto Camassa, Richard M. McLaughlin bernardi@live.unc.edu

The "Taylor Pipe Flow" experiment was designed to be a continuation of the research on the dispersion of soluble matter through a tube conducted by G.I. Taylor in the '50s. In two-dimensional channel models and three-dimensional model glass pipes with circular or square cross-sections, we explore the theory of Taylor dispersion, explaining the motion of a passive scalar transported by laminar flow. Studies at the University of North Carolina at Chapel Hill are implemented analytically, numerically and experimentally to better understand the evolution of the dispersion of solute, primarily by calculating the first four moments of its concentration, leading to the computation of variance, skewness and kurtosis. Our experimental setup allows us to observe the effects of Poiseuille flow as either advection or diffusion dominates in different regimes, characterized by the Taylor time scale $t_T \propto a^2/\kappa$, depending on the characteristic length and the diffusion coefficient. We conduct experiments to better understand these regimes, characterized by the dimensionless Péclet number, $Pe = ua/\kappa$, where a is the pipe radius, u is the characteristic velocity, and κ is the diffusion coefficient of the solute. Experimentally, we take the intensity of a fluorescein-dyed portion of distilled water and find its corresponding concentration by solving an inverse problem of intensity to concentration. This serves as results to compare with the theoretical approach. Such experimental analyses are done keeping in mind possible physical applications ranging from the smallest microscales, in drug delivery via capillary blood flow, to the largest scale, in distribution of contaminants in rivers and estuaries.

2:05 *Combustion Waves and Wave Sequences in Porous Media*

Fatih Ozbag NCSU Advisor: Stephen Schecter fozbag@ncsu.edu

In this work we study combustion waves that occur when air is injected into a porous medium containing initially some fuel. Using phase plane analysis, we prove the existence of various combustion waves for a system of three partial differential equations that give temperature, oxygen and fuel balance laws. We also study the spectrum of the linearized system at a traveling wave and introduce weight functions to move the spectrum to the left half-plane. Moreover we list all possible generic wave sequences that solve boundary value problems.

2:30 *Asymptotic Preserving Schemes for Kinetic Chemotaxis Equations*

Seyma Nur Ozcan, , Alexander Kurganov, Maria Lukacova NCSU Advisor: Alina Chertock snozcan@ncsu.edu

"Chemotaxis models are used to describe the movement of cells in response to the chemoattractant in a medium. The most common models for this phenomenon are the Keller-Segel equations, which can be derived as the drift-diffusion limits of kinetic equations. These diffusive limits are obtained from the nondimensionalized form of kinetic equations by using a parabolic scaling. In this talk, I will present an asymptotic preserving scheme for the kinetic chemotaxis models, in which the numerical scheme that solves the kinetic equation leads to the scheme of the limiting equation.

Modeling, SAS 2106

1:15 *Constructing Investor Views on Black-Litterman Model*

Cagatay Karan, Tao Pang NCSU Advisor: Tao Pang `ckaran@ncsu.edu`

"The Black Litterman Model (BLM) has contributed to modern portfolio theory a new perfective where the investor views are combined with historical estimates. The resulting portfolio usually has a better performance than the solution obtained in the classical mean-variance model. We will show our results on constructing investor views using Mixed Gaussian distribution.

1:40 *Three-dimensional refraction correction for contact lens metrology using SDOCT imaging*

Micaela Mendlow NCSU Advisor: Mansoor Haider `mrmendlo@ncsu.edu`

Spectral domain optical coherence tomography (SDOCT) is a non-invasive, high resolution medical imaging technique based on near-infrared interferometry. Its ability to quickly and safely produce detailed cross-sectional images of biological microstructures has made SDOCT a popular tool in ophthalmology. A potential new application of SDOCT is off-eye validation of contact lens design parameters. In collaboration with Bioptigen Inc., we are working to develop algorithms that will generate an accurate three-dimensional representation of a contact lens using images obtained via their Envisu S4410 SDOCT system. Because the light waves used in SDOCT are subject to refraction in optical media, images of interior surfaces appear warped. This warping is typically corrected by applying Snell's Law to each cross-sectional image independently, relying on the unlikely assumption that the refracted light ray travels within the same plane as the image. In this presentation, we will discuss the development and implementation of a three-dimensional refraction correction model derived from principles of geometric optics.

2:05 *Geometry of the Null Penrose Inequality*

Henri Petrus Roesch Duke Advisor: Hubert L. Bray `henri@math.duke.edu`

Given an embedding of a topological sphere into an ambient spacetime we are able to define its Hawking mass. This quasi-local energy functional has been fundamental in proving special cases of the Penrose inequality which conjectures that the square total mass of a spacetime be bounded below by the total area of its black holes. Our interest lies in studying this problem from the perspective of null cones emanating from topological spheres, for which the Hawking mass provides information regarding the energy bounded within. In this talk we will make extensive use of a simple model for a single black hole to motivate some recent results.

2:30 *Geometric Skewness in the Passive Tracer Problem*

Manuchehr Aminian, Francesca Bernardi, Roberto Camassa, Richard McLaughlin UNC-CH Advisor: Roberto Camassa, Richard McLaughlin `aminian@live.unce.edu`

The classic work by G.I. Taylor describes the enhanced longitudinal diffusivity of a passive tracer in laminar pipe flow. Much work since then has gone into extending this result particularly in calculating the evolution of the scalar variance. However, less work has been done to describe the asymmetry of the distribution. We present the results from a modeling effort for the general picture of how the higher moments of the tracer distribution depend on geometry. We do this via analysis of "channel-limiting" geometries (rectangular ducts and elliptical pipes parameterized by their aspect ratio), using both new analytical tools and Monte-Carlo simulation, which have revealed a wealth of nontrivial behavior of the distributions at short and intermediate time.